The Relationship between Short-Run Pricing and Investment Timing: The Port Pricing and Expansion Example

Jack Devanney and L H Tan
Center for Tankship Excellence, USA, djw1@c4tx.org

Abstract

A dynamic program for generating monopoly profit-maximizing pricing and expansion policies for a port facing a continuously shifting, price-dependent demand and a series of discrete investment alternatives is described. A related algorithm for generating the societal income maximizing policy is presented. These models demonstrate the coupling between short-run pricing and investment timing. In contrast to much of the literature in the area, they argue that under all but pathological situations, there is no "structural" conflict between short-run allocative efficiency (marginal cost pricing) and long-run allocative efficiency (attracting the proper level of capital).

1 Introduction

The relation between the necessary conditions for the efficient short-run allocation of resources (marginal cost pricing) and the necessary conditions for long-run efficiency (attracting the appropriate levels of capital) has long been a bone of contention. In the past, it has often been alleged that in a number of situations of practical interest, the two principles are inconsistent. This argument has been applied with pernicious effect in the marine transport industries, among others. Those who have argued that the principles governing short-run efficiency and long-run efficiency are not inconsistent, while to our mind entirely persuasive, have not, in our opinion, placed their arguments on firm quantitative foundations.

The purpose of this paper is to offer a demonstration based on a reasonably general model that for all but entirely pathological situations, short-run and long-run allocative efficiency are not only not inconsistent, but are intimately and necessarily tied together. A by-product of this demonstration is a quantitative method for both the short-run and long-run regulation of monopolies. There has been much confusion in this area, once again under the impression that marginal cost pricing will lead to less than normal return on investment in the face of large fixed investments.

The vehicle which we will use to present our arguments is port pricing and expansion. This is a product of the authors' particular research interests. Fortunately, however, this example combines all the elements required to demonstrate how short run pricing and timing and level-of-investment decisions can be coupled to generate both short-run and long-run efficiency. The translation to other areas of application will be obvious to the reader. Once cautionary note: the argument which we will develop assumes that changing prices is costless. In situations such as urban mass transit management, this hypothesis may require some modification.

We will begin with a situation in which the interaction between short-run pricing and investment timing is particularly clear-cut: port pricing and expansion under the objective of maximum private profits. While we hold little brief for this particular objective, this problem will serve to demonstrate the basic line of reasoning which will be used throughout. Secondly, we will move to a delineation of port pricing and expansion under the objective of maximum world income — more precisely, Pareto-optimality with respect to the prices prevailing outside the port. Thirdly, we will generalize the results to a multi-commodity port to demonstrate that joint costs represent no problem for our basic reasoning.

2 Monopoly Profit Maximization

The first objective function which we will examine in monopoly profit maximization. For a variety of reasons, the operation of a particular port almost always ends up under the control of a single entity, usually some public body. Such centralization implies that the controlling body has a degree of monopoly power over the shippers and hinterland that it serves. Thus, a possible objective function for such a port is to operate in such a manner as to maximize the present value of the differences between its revenues and its outlays.

Our investigation of this objective function does not necessarily mean that we recommend it, nor that it is a common objective among actual ports. Quite the contrary is true in both cases. It implies only that we feel that analysis of this objective will lead to some useful insight.

2.1 The Basic Model

In analyzing all our objective functions, we will consider the following extremely simple port:

1. The port offers a *single*, homogeneous cargo handling service. That is, we might imagine a completely specialized port which handles only one commodity. The amount of cargo-handling services performed by the port in some time period, n, can be measured by the throughput of this commodity in this period, x_n, in say, tons. Supposing a non-discriminating monopolist, the port's pricing policy through time can also be described by a single number, p_n, in, say, dollars per ton. For exposition's sake, we will assume the period in question is a year, although it could just as easily be a month or a season. Further, we will assume that the period is short enough so that the port is willing to act as if its demand curve is constant over this period.
2. At discrete points in time, say once a year, the port has the opportunity to expand. However, the port has only one such expansion opportunity at such a time. To wit, it can increase the design capacity, C_n of the port by ΔC tons by making an outlay of $EC(C_n)$ dollars, or it can choose to make no change in design capacity at this time.[1] That is, we might imagine a port whose only expansion alternative is, once a year, to add another berth of design capacity, ΔC. We will assume that, if the port decides to expand at the beginning of the nth period, t_n, the berth will become available at the end of that period. We will also assume that any such investment will last forever.[2]
3. Let $VC(x_n, C_n)$ be the throughout-dependent expenses associated with moving a quantity x_n in period n given an installed design capacity of C_n at that time. We will assume that VC is a non-increasing function of C_n and that its derivative with respect to x_n, $MC(x_n, C_n)$ is a non-decreasing function of x_n. For ports, *for a given* C_n, marginal cargo handling cost is generally constant up to some level, whereupon it increases sharply, finally becoming vertical at the point where it is impossible to further increase throughput. At this point, the marginal cost to the port of handling a unit of cargo becomes the maximum that a turned-away unit of cargo would have been willing to pay for this service. Thus, our concept of marginal cost includes the "congestion cost" of Allais (1964).
4. Finally, we will assume that the demand for the port's service in period n, $D(p_n, t_n)$ is a function only of the price in that period and time. In many respects, this is the most limiting assumption of all. In real life, the port's pricing policy through time will affect the long run growth in demand either though long-run adjustments by shippers or by encouraging the development of competing ports whose existence will then affect the demand perceived by the monopolist. Our assumption that the *growth* is demand is unaffected by past pricing policies rules out these phenomena.

2.2 A dynamic program for obtaining the optimal pricing-expansion policy

We will assume that the port's cost of capital is constant at $r\%$ per annum denote the associated discount factor by ρ. The demand surface through the future, $D(p, t)$ is known and the monopolist is willing to assume that demand is constant through an individual period — a year in our case. That is, he is willing to act as if demand makes a discrete shift to the right at the end of each period and then remains constant through the ensuing period. This implies that the short-run profit maximizing price will be constant through an individual period.[3]

At the beginning of the nth period, t_n, the port's current situation is completely described by the amount of design capacity already installed, C_n. Define $V_n(C_n)$ to be the maximum present-valued profits obtainable from t_n on, if at t_n the port has C_n units of design capacity operating. V_n is the present value of future profits, present valued back to t_n. At t_n, in this situation, the port has two decisions to make:

1. How much should it charge for its service for the period t_n to t_{n+1}?.
2. Should it order an expansion of ΔC at t_n or not?

Given a particular C_n *at* t_n, the two decisions can be separated, for any new expansion ordered at t_n will not become available until t_{n+1}. In this short run

[1] More precisely, the expansion cost, $EC(C_n)$, is the present value of the time stream of expenses to which the port commits itself when it decides to make the expansion, including any maintenance costs that are independent of throughput.

[2] Finite investment life can be accommodated by the basic reasoning we will use without conceptual difficulty. However, finite life involves some rather severe computational problems as the state space becomes very large.

[3] This requirement can always be met by simply making the length of an individual period short enough. In port problems, one will rarely have to go to a period of less than a quarter and in many cases a period of a year or more will suffice.

situation, a classical monopolist will maximize his short run profits by setting price such that marginal revenue equals marginal cost, that is, by solving the equation

$$\frac{\delta}{\delta x}\left[D^{-1}(x^*(C_n,t_n),t_n)x^*(C_n,t_n)\right] = MC(C_n, x{*}(C_n,t_n)) \quad (1)$$

for $x^*(C_n,t_n)$ where D^{-1} is the inverse of the demand function. $D^{-1}(x{*}(C_n,t_n))$ is the monopolist's profit maximizing price, $p^*(C_n,t_n)$ in this situation, and the resulting maximum net operating revenues for the period (t_n,t_{n+1}) are

$$R^*(C_n,t_n) = p^*(C_n,t_n)x^*(C_n,t_n) - VC(C_n, x{*}(C_n,t_n)) \quad (2)$$

Equation 2 holds whether or not the port decides to expand at t_n given C_n because of the construction delay.

Of course, price is not the only variable under the port's control. It also has control over the amount of design capacity installed. Using static analysis, it is well known that a monopolist will maximize his long run profit by investing in the amount of capacity such that *when he charges the short-run profit maximizing price*, he will be operating at the minimum point on his average cost curve.

The problem is that the port is not faced with a static situation. Typically, the demand for the port's services will be growing, which means that the demand curve will be continuously shifting to the right through time.[4] In order to match this growth, the port would have to be continuously shifting the amount of design capacity. Unfortunately, in transportation design capacity generally only comes in discrete chunks. It is usually not useful to consider building half a berth or buying half a crane. To illustrate this problem, we have assumed that our example port has only one expansion option: once each year it may purchase ΔC units of design capacity or none at all. There is no in between.

Examining these options: if the port decides not to expand at t_n given C_n, then the maximum present valued profits obtainable through the future present valued back to t_n are

$$R^*(C_n,t_n) + \rho V_{n+1}(C_n) \quad (3)$$

If on the other hand, the port chooses to expand at t_n, then the present value of future profits assuming optimal operation from t_{n+1} on is

$$R^*(C_n,t_n) - EC(C_n) + \rho V_{n+1}(C_n + \Delta C). \quad (4)$$

The monopoly profit maximizing port will choose the maximum of these two options. Hence we have the following recursion relation,

$$V_n(C_n) = max\begin{cases} R^*(C_n,t_n) + \rho V_{n+1}(C_n) \\ R^*(C_n,t_n) - EC(C_n) + \rho V_{n+1}(C_n+\Delta C) \end{cases} \quad (5)$$

which holds for all possible values of installed capacity, C_n, and for all possible $n = 0,1,2,3,...$; that is, for all possible decision points, t_n.

In order to be able to numerically solve this set of equations, we must assume a boundary condition on V_n at some time in the future. One such boundary condition follows from supposing that at some time in the relatively distance future, t_N, demand will cease to grow, in which case it will be optimal not to order any expansion after t_N, nor will the profit maximizing price change.

Let $R{*}(C_N,t)$ be the resulting profit maximizing revenue obtainable in any a period for which $t > t_N$ given that the installed capacity from t_N on is C_N. Since this amount is constant through the future from t_N on, the present value from t_N given C_N is

$$V_N(C_N) = \frac{R^*(C_N,t_N)}{1-\rho} \quad (6)$$

yielding the boundary condition at time t_N in the future for all C_N. Starting with this boundary condition and employing backwards recursion, one can solve for the optimal value function for all $V_N(C_N)$ and the corresponding profit-maximizing expansion and pricing policy.

2.3 A Sample Problem

A computer program implementing the above dynamic program has been written. We have exercised it on the following sample problem.

1. Demand linear in price with exponentially decreasing growth

   $$D(p,t) = (1 - e^{-\gamma t})(\alpha - \beta p) \quad (7)$$

   For all our sample exercises in this paper, we have held the demand surface constant, setting $\alpha = 10^6$ tons, $\beta = 10^4$ tons per dollar, and $\gamma = 0.1$. This demand surface is shown in Figure 1. For this demand surface, price can run between \$100 per ton and \$0 per ton, and the resulting throughput will be between 0 tons and a number which is 0 at $t = 0$ but fairly rapidly approaches 1,000,000 tons per year as t approaches 40 or so. For this demand surface, we have taken t_N to be 50 since practically all the growth has taken place by this time.
2. Marginal costs of each berth are identical and quadratic in throughput. Given identical marginal costs, the monopolist will distribute his throughput, x, evenly among each of the berths. Thus, at any time, x/I tons of cargo

[4] The shift to the right in demand may have seasonal fluctuations superimposed on it which may temporarily move the demand curve to the left. As long as the general trend is to the right, such fluctuations present no problems for the analysis that follows.

will be flowing through each berth, where I is the number of currently installed berths, $C/\Delta C$. The marginal cost function which was used in the sample problems was

$$MC(x, C) = \frac{3EC(1-\rho)}{2\Delta C^3}(x/I)^2 \qquad (8)$$

where the constant $1.5EC(1-\rho)\Delta C^3$ has been chosen to make the average cost curve minimum when throughout equals design capacity.[5]

This simple structure was chosen because it makes interpretation of the results easy. The basic algorithm can accept any demand function and cost structure meeting our rather general conditions.

2.4 Results

The results of these sample calculations for $\rho = 0.9$, $EC = 10^6$ and $\Delta C = 10^5$ tons per year, $50,000$ tons per year and $25,000$ tons per year are shown in Figures 2, 4 and 6 respectively. There are several things to notice about these figures. One is that price varies very little from the marginal revenue-maximizing price, which is always \$50 per ton for our rather strange demand growth pattern. This is due to the much lower marginal costs. Except for Figure 6, where the monopolist cannot bring on capacity as fast as he would like, the price is practically unaffected by the present situation, remaining in the neighborhood of \$51 to \$53 throughout. The corresponding marginal cost is in the neighborhood of \$2 or \$3, except for Figure 6, where it moves to about \$7 per ton.

The profit maximizing monopolist alternates periods in which design capacity is higher than throughput with periods in which the reverse is true, *adjusting price downward every time he increases capacity.* It does not always pay the monopoly profit maximizer to delay expansion to the point where the expansion is immediately utilized at design capacity. The number of berths approximately doubles with each halving of the design capacity of each berth. As a result the total throughput and the final situation are quite similar in each case, as is necessarily the case given the similarity in the prices charged. This occurs despite the fourfold increase in costs from Figure 2 to Figure 6, and the fact that no one would regard the demand surface of Figure 1 to be particularly inelastic. The reason why design capacity is not quite equal to throughput at steady state is that the port has only a finite number of design capacities available and the thus the monopolist (program) is forced to choose the "closest" of the design capacities available. Given the parameters chosen, the port is a very profitable enterprise for all three cost structures.

In order to investigate behavior in a situation where the monopolist could not make as much money, we reran these three cases multiplying EC by 10. That is, a berth now costs \$10,000,000. The results are displayed in Figures 8, 10 and 12. In these situations, the constraint of no more than one new berth per year is never limiting; the general level of his price is fairly constant throughout the period for each ΔC. However, since the monopolist no longer compensates for a having of ΔC by doubling the number of berths, the level of this price now changes markedly with a change in ΔC with a resultant effect on throughput. Notice that with the lower ΔC's the penalty for off-design operation is so high that it pays the monopolist to stick quite close to design capacity throughput. Optimal profits drop from \$81,000,000 for $\Delta C = 100,000$ to \$15,000,000 for $\Delta C = 25,000$ Another sizable increase in EC would undoubtedly make the port an unprofitable investment for the monopolist. at least for the smaller ΔC's; that is, he would never invest in the first berth.

If one keeps EC at 10^7 but increases the sample design capacities to 5,000,000 tons per year, 2,000,000 tons per year, and 1,000,000 tons per year respectively, the the monopolist buys one berth, and his corresponding present valued profits are, in each case, slightly in excess of \$106,000,000, and the profit maximizing prices stays within 20 cents of \$50 per ton throughout. The reason for mentioning this particular set of parameters will become clear when we compare these results with the corresponding results for an economically efficient port in the following section.

3 Societal Income Maximization

The second objective function which we wish to investigate for the same port is economic efficiency. Assuming all prices exogenous to the port equal marginal social costs,[6] then the necessary conditions for maximum societal income are:

1. In any short-run situation, the port must charge the marginal social cost for its services.
2. The port should expand as soon as the capital (the resources) required for the expansion is more valuably employed in the the port than elsewhere.

Several authors have intimated that these two principles are contradictory when one is faced with large, indivisible capital investments. We shall see that as long as, for the *smallest* possible level of investment, the average cost curve *eventually* turns upward (as it must when the investment is operating at greater

[5] This is a purely expositional convenience. The entire line of reasoning does not depend on the concept of "design capacity" in any fundamental way. Nor does it depend on "average cost", which strictly speaking applies only to the steady-state situation.

[6] In the real world, the situation is considerably complicated by the cartelization of the liner trades. In this situation, a decrease in cargo handling cost may and has been appropriated by the liner conferences, whose freight rates include cargo handling (see Devanney, Livanos, and Stewart, 1972). We will conveniently ignore this problem.

than design capacity), not only are these two principles not contradictory, they are essentially and necessarily tied together.

Analysis of the short-run situation follows directly from the marginal cost pricing principle. If at the beginning of the nth period, t_n, the port has C_n units of design capacity installed and the demand for the port's service is $D(p, t_n)$, then the economically efficient price for the period (t_n, t_{n+1}) is given by solving

$$D^{-1}(x^*(C_n, t_n), t_n) = MC(C_n, x * (C_n, t_n)) \tag{9}$$

for the economically efficient throughput, x^*. The economically efficient price, $p^*(C_n, t_n)$ is equal to $D^{-1}(x * (C_n, t_n), t_n)$.

Application of the second principle is slightly less straightforward. We suppose a perfect capital market, and let the social cost of capital be r% per year. Let the corresponding discount rate be ρ. Then the second principle says the port should expand as soon as the present value of the earnings of the expansion, *where these earnings result from the above marginal cost pricing philosophy*, net of outlays associated with the expansion is positive when discounted at an interest rate r.

The problem is that, given the coupling between pricing and expansion implied by marginal cost pricing, the future earnings of a berth constructed now depend on the expansion alternatives followed in the future. Thus, in order to tackle the expansion problem, we must, as in Section 2, work backwards from the far distant future, figuring out what expansion alternative will be followed for every possible situation the port might get itself into.

At this point, we will make one addition assumption: each additional unit of capital investment — each additional berth, if you will — is exactly similar to the berths already in operation as far as the shipper is concerned.[7] Each berth performs the same service with the same marginal costs.[8] In such a situation, and once again assuming marginal costs are non-decreasing, if $x_n(C_n, t_n)$ is the total throughput for the port in the nth period, common sense ans symmetry suggest that efficiency requires that the throughput be divided equally among the berths. Each berth will handle $x(C_n, t_n)/(C_n/\Delta C)$ units of cargo. Also of course the price charged for this service will be the same at each berth during this period. Under marginal cost pricing, the net operating revenues of *each* the $(C_n/\Delta C)$ installed berths in the nth period will be

$$r^*(C_n, t_n) = \frac{p^*(C_n, t_n)D(p^*, t_n) - VC(C_n, x * (C_n, t_n))}{C_n/\Delta C} \tag{10}$$

If this is the case, we can define $W_n(C_N)$ to be the present value of the earnings net of variable costs of a berth from t_n on, if at t_n, C_n units of design capacity are installed, and if from t_n on, we follow an economically efficient pricing and expansion policy. Thus, W_n refers to the future operations of any one of the already installed berths present valued back to t_n

The job before us, then, is to develop a recursive method for computing $W_n(C_n)$. As in Section 2, we will start from some time in the relatively distant future, t_N, where demand is no longer growing, and therefore no further port expansion is ordered, In this steady state situation, we have

$$W_N(C_N) = \frac{r^*(C_n, t_n)}{1 - \rho} \tag{11}$$

for $r^*(C_N, t_N)$ will be earned by each berth in each period from t_N on. This relation yields $W_N(C_N)$ for all possible values of the design capacity at t_N.

Now lets consider the situation at t_{N-1} given some installed capacity C_{N-1}. The net present value of an additional berth ordered at t_{N-1} in this situation, $V_{N-1}(C_{N-1})$ is made up of the net operating revenues this berth will earn from t_N on less the present value of the expansion costs to which the port commits itself when it orders the berth, or

$$V_{N-1}(C_{N-1}) = -EC(C_{N-1}, t_{n-1}) + \rho W_N(C_{N-1} + \Delta C). \tag{12}$$

where $V_{N-1}(C_{N-1})$ is the net present value of the investment present valued back to the time at which it is ordered, t_{N-1}.

Following the second basic principle of efficient port pricing ans expansion, the expansion should be ordered if $V_{N-1}(C_{N-1}) \geq 0$; otherwise it should not. Let $e^*(C_{N-1}, t_{N-1})$ be 1 if the efficient choice in this situation is to expand and 0 otherwise. In order to move back to t_{N-2}, we must first compute the earnings of a berth from t_{N-1} on by

$$W_{N-1}(C_{N-1}) = r^*(C_{N-1}, t_{N-1}) + \rho W_N(C_{N-1} + e^*(C_{N-1}, t_{N-1})\Delta C) \tag{13}$$

Notice the future earnings of a berth which is already installed at t_{N_1} depend on our expansion choice at t_{N-1} since this expansion will change both the throughout and the price at each berth. Having computed W_{N-1}, we can compute

$$V_{N-2}(C_{N-2}) = -EC(C_{N-2}, t_{n-2}) + \rho W_{N-1}(C_{N-2} + \Delta C). \tag{14}$$

Once again, if $V_{N-2}(C_{N-2}) \geq 0$, $e^*(C_{N-2}, t_{N-2}) = 1$; otherwise $e^*(C_{N-2}, t_{N-2}) = 0$. And

[7] We also need a requirement which is the dynamic equivalent of non-decreasing long-run average costs. This will be discussed in more detail later.

[8] This assumption is made for the purposes of expositional convenience only, although other assumptions will lead to some computational problems (excessive memory requirements).

$$W_{N-2}(C_{N-2}) = r^*(C_{N-2}, t_{N-2}) + \rho W_{N-1}(C_{N-2} + e^*(C_{N-2}, t_{N-2})\Delta C) \quad (15)$$

At this point we can move back to t_{N-3} and repeat the process. Working our way backwards, we can construct the entire efficient expansion table $e*(C_n, t_n)$ for all possible combinations of C_n and t_n. We can then move forward through this table, starting at the present, t_0, with the present installed capacity, C_0 picking out the economically efficient policy, Once one has the efficient expansion policy, it is an easy matter to recompute the sequence of short-run prices and corresponding throughputs using marginal cost pricing.

Notice that this pricing and expansion policy has a very interesting property. Although at any time we follow strict marginal cost pricing based only on short-run capacity and short-run demand, the port as whole over its life does not lose money. It will not require a subsidy. The periods of under-utilization (throughput less than design capacity [price less than average cost]) and congestion (throughput greater than design capacity [price greater than average cost]) work out so the entire present-valued time stream of revenues just covers the entire present valued time stream of costs. This is required if long-run allocation is to be efficient.

In 1938, Hotelling in the process of advocating strict marginal cost pricing, suggested that "congestion charges" might cover losses in areas where marginal costs were less than average costs. This suggestion has come in for considerable criticism (Ruggles, 1949-1950), and a good part of the literature on marginal cost pricing has dealt with marginal cost pricing's supposed requirement of subsidies (Vickrey, 1955). However, the above analysis indicates that, under certainty, marginal cost pricing coupled with efficient investment not only *could* result in full costs being covered, but *must* so result. *Efficiency through marginal cost pricing and full cost recovery not only are not inconsistent — they are intimately and necessarily tied together.* Some writers have been misled by the persistant over-capacity generated in many markets where marginal cost pricing is not followed. If one wishes to see how marginal cost pricing operates, one should turn to the truly competitive markets.

The tanker charter market is an example. At any point, the spot charter rates equals the marginal cost of the marginal ship. In periods of over-tonnaging, this rate is well below the average cost of the average ship. In booms, it is well above. In the long run, the fluctuations in marginal cost of the marginal ship average out to the average cost of the marginal owner. Thus, this market has no problem atttracting capital in the long run, despite the fact that the out-of-pocket expenses of the average owner of supplying the transport services (fuel and port charges) are less than 20% of all his expenses.

Essentially, all the algorithm does is simulate this competitive market dynamic. However, it does so in a somewhat more systematic manner than the trial-and-error process used by actual competitive markets. Under certainty — and as long as society is willing to act as an expected value decisionmaker, uncertainty can be incorporated without difficulty[9] — neither the competitive market nor the algorithm can hope to follow a policy which, given hindsight, is unimprovable. However, the algorithm can avoid one sort of error which certain competitive markets are prone to, and that is the phenomenon where all suppliers read the present situation as profitable and decide to expand without accounting for the impact of this total expansion on future prices. Chastened by the results, they become overly conservative. This process, combined with construction lags and growing periods, leads to a certain excess jerkiness in some markets' operations, which in turn can lead to unduly prolonged losses (or profits), calls for subsidy, protection, regulation, attempts at cartelization, etc.

Nevertheless, the potential utility of the algorithm is not in replacing competition in those markets where it has been maintained, but rather in substituting for competition in those markets where it has not been maintained. The above competitive process, however jerky, cannot, assuming free entry and exit, stray too far away from allocative efficiency for too long. However, in those situations where it does not pay to maintain multiple suppliers in order to keep the competitive market dynamic going, or, more widely, in situations where the institutions for implementing the dynamic have not developed or have been suppressed, an artificial means of simulating this process is required. The above line of reasoning can serve as such a substitute. Of course, all kinds of institutional difficulties confront the implementation of such an algorithm. For certain transport technologies, e.g. airports, the algorithm may imply a long period of deficits followed by a period of surpluses. Disciplining a public body to charge surplus-producing prices after a history of running deficits presents some obvious problems.

In view of these practical difficulties, maybe the best we can expect from this reasoning is an end to the common rejoinder to pleas for marginal cost pricing: "Look I've got decreasing average costs. You economists have proven marginal cost pricing is infeasible in my case."

[9] One must assign probabilities to the future such that the probability of each uncertain demand variable at t_{n+1} depends only on the state of things at t_n. One then replaces V_n and W_n by their expected values and proceeds exactly as before to generate the efficient expansion table. However, an additional state variable will be required for each random variable, and the resulting policy will, like the actual market, be adaptive in the sense that the decision actually taken at some time, t_m, in the future will depend on what has happened between now and t_m. One can also handle constant risk aversion in an analagous manner. See Devanney (1973).

3.1 Results for sample problem

A program for implementing the above algorithm has been written and exercised on the sample problem of Section 2. The results are displayed in Figures 3, 5 and 7 for exactly the same demand and cost structures for which the monopolist's optimal policies are shown in 2, 4 and 6.

In each case, the number of berths installed and the throughput is about double the respective monopoly profit maximizer's policy. For $\Delta C = 100,000$ (Figure 3), the port has no trouble keeping up with the early stages of demand growth and marginal costs and price is always less than \$2.00, approximately 1/25 of the monopolist's optimal price. Interestingly, the efficient policies tend to operate at higher utilization than the monopolist, always dropping price so that throughput is equal to or greater than design capacity. To put it another way, for this demand and cost structure, the efficient port doesn't expand until (under its pricing policy) the new capacity will be fully utilized. As a result, the steady-state solution involves throughputs slightly higher than design capacity, since in order to fully utilize further expansion, price would have to be dropped below marginal cost. (The postulated demand surface is extremely inelastic for prices of less than \$5.00.) One result of this is that in Figure 3, the port ends up making a very slight profit.

In Figure 5, the port's expansion is clearly limited by the constraint that it can only expand once a year. As a result, efficient allocation of the available capacity involves prices of up to \$10.00 before expansion is able to catch up. A much more severe case of this situation is shown in Figure 7, where expansion is unable to catch up until $t = 35$. Efficient pricing for this situation involves marginal costs of up to \$39.00. As a result of the expansion constraint, the ports in Figures 5 ans especially 7 make substantial profits. From the point of view of societal income, this is a bad sign. It indicates that, if we were to relax the constraint that only one berth can be constructed per year, an efficient expansion policy would take advantage of this relaxation. However, given the constraint, the port must allocate its scarce resources efficiently. Hence the high prices.

In general, the efficient port's prices are a good deal more variable than the monopolist's, making the point that efficient allocation calls for considerably more price flexibility than the monopolist — and most port administrators — care for.

Of course, both the monopolist's and the economically efficient pricing policies involve more flexibility than typical average cost pricing, however defined. And both involve decreases in price immediately after an expansion, while average cost pricing involves either no change or an increase in price, depending on the accountant's degree of allegiance to the past.

In Figures 9, 11 and 13, we have increased EC to \$10,000,000, creating the situations equivalent to Figures 8, 10 and 12. Under this considerably more adverse cost structure, for a ΔC of 100,000 tons per year, the port delays expansion until it is operating approximately 10% over design capacity. As a result, marginal cost and prices fluctuate in the neighborhood of \$17. Halving the design capacity, Figure 11, results in approximate doubling of the efficient marginal costs, which are now high enough to have a significant effect on throughput. That is, the economically efficient port no longer doubles the number of berths with halving of the design capacity. This phenomenon becomes even more pronounced with a further halving of design capacity in Figure 13. Despite this decrease in ΔC, the port buys no more berths than in Figure 11, although it brings them on a little sooner. As a result, marginal costs approximately double and prices begin to approach those charged by the monopolist in a similar situation. The corresponding throughput is halved. Notice that the port is still operating at little more than 10% over design capacity. The marginal cost curve at this point is very steep, so the efficient port chooses to increase price rather than *try* to push more throughput through these low capacity berths. This behavior contrasts rather sharply with the typical real-life response to limited or expensive capacity.

If one maintains EC at \$10,000,000 but increases ΔC to 1,000,000 tons per year, 500,000 tons per year, and 250,000 tons per year, an interesting phenomenon occurs which demonstrates a fundamental limitation on the above line of reasoning. For $EC = 10^7$ dollars and $\Delta C = 10^6$ tons per year, the program implementing the above algorithm chooses not to build any berths. However, when ΔC drops to 500,000 tons per year, the algorithm buys one berth and runs 37,000,000 tons of cargo through it over the life of the port. At first glance, this would seem to imply that if our cargo handling technology becomes good enough, we don't handle any cargo, The problem is that with a ΔC of 1,000,000 tons per year and the sample demand surface, it is impossible even with only one berth to get on the up side of the average cost curve even when demand has reached its full growth. If the investor invests at all, he is faced with *unavoidable* decreasing average costs at full demand growth. The basic condition for long-run stability of a competitive market is violated. Notice that it is *not* necessary that we not have decreasing average costs throughout the process. *We require only that we not have decreasing average costs after full demand growth has been reached for the smallest possible investment.* In actual fact, one would rarely run into a situation where the smallest possible investment resulted in throughputs less than design capacity at full demand growth. When one views the problem dynamically, the condition for the operation of a competitive market is much weaker than some interpretations of static analyses would have us believe. Unavoidable decreasing average costs present no problem for the monopoly profit maximizer, as indicated by the last paragraph of Section 2.

4 The Multi-Commodity Port

Exactly the same line of reasoning for the objective functions of both Sections 2 and 3 can be applied to the multi-commodity port.[10] Consider the same port as before, except that now it is handling a number, M of different commodities. Let x_i and p_i be the throughput and price charged on commodity i in some period. Let $X = \{x_i, x_2, \ldots, x_M\}$ and $P = \{p_i, p_2, \ldots, p_M\}$. In general,the demand for the ith good, $D_i(P,t)$ will depend on all the short-run prices and time, and the variable cargo handling costs will depend on all the throughputs and the installed capacity, $VC(X, C_n)$. We assume $\delta D_i/\delta p_i < 0$ and $\delta VC/\delta x_i \geq 0$.

For the private profit maximizer, at any decision point t_n given C_n, the requirement that marginal revenue equals marginal cost leads to the standard set of equations:

$$\frac{\delta}{\delta x_i}(D^{-1}(X,t_n)x_i) = \frac{\delta}{\delta x_i}VC(X,C_n) \tag{16}$$

for $i = 1, 2, \ldots, M$, which can at least conceptually be solved short-run profit maximizing throughputs $X^*(C_n, t_n)$ and prices $P^*(C_n, t_n)$. From this point on the dynamic program looks exactly like that in the one commodity case

$$V_n(C_n) = max \begin{cases} R^*(C_n,t_n) + \rho V_{n+1}(C_n) \\ R^*(C_n,t_n) - EC(C_n) + \rho V_{n+1}(C_n + \Delta C) \end{cases} \tag{17}$$

where $R*(C_n, t_n) = P^*X^* - VC(X^*, C_n)$ with essentially the same boundary condition.

For the societal income-maximizing port, the short-run throughputs and prices for t_n and C_n are given by the solution of

$$D^{-1}(X,t_n) = \frac{\delta}{\delta x_i}VC(X,C_n) \tag{18}$$

for $i = 1, 2, \ldots, M$, after which the algorithm is exactly the same as before.

The only point of this rather uninteresting drill is that there is no more need to "allocate" "joint costs" across commodities or services than there is to allocate "fixed costs" across the same service performed at different times. The "joint cost problem" disappears as soon as one makes one's objective function and options *through time* explicit. It is interesting to note that the fact that we have assumed that the capital investment is completely "joint" simplifies rather than complicates the dynamic program from a computational point of view. For example, if there were M types of berth, one for each type of cargo, but either variable costs or demand were interrelated, then the dynamic program would require M state variables and the size of the state space would increase combinatorially.

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[10] Also by combining this reasoning with a bit of optimal tariff theory, one can generate and algorithm for port pricing and expansion which maximizes the income of a subset of society such as the nation in which the port is located. See Devanney and Tan (1973).

Figure 1: Example Demand Surface

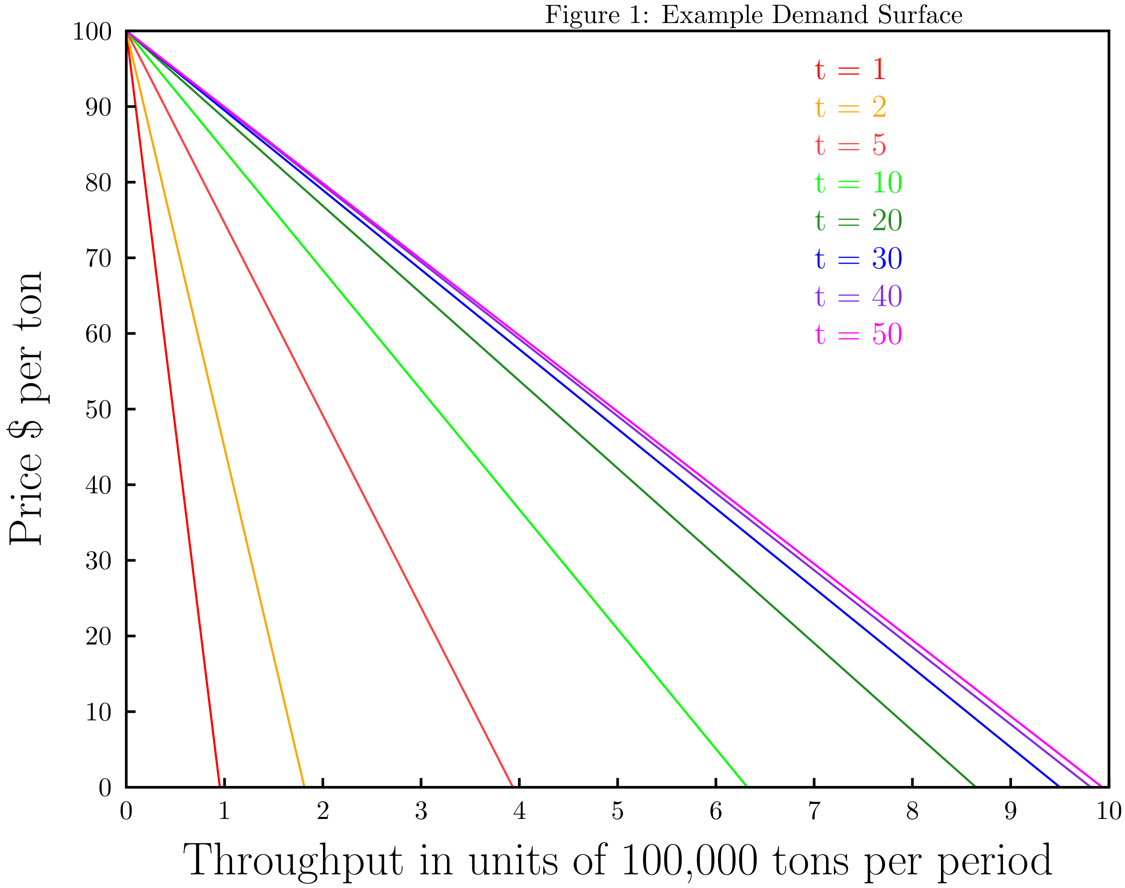


Figure 2: Monopoly Profit Maximizing Policy, $\Delta C = 10^5$, $EC = 10^6$

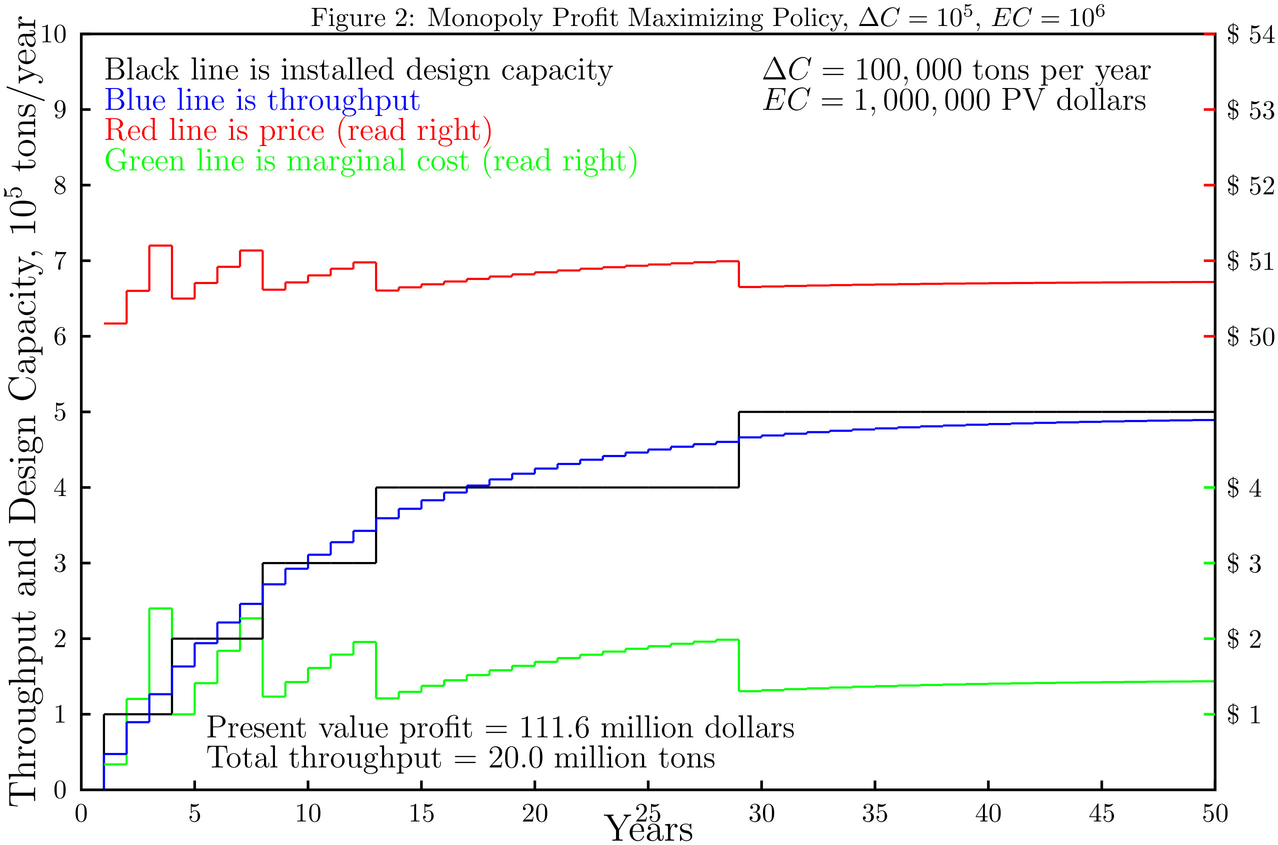


Figure 3: World Real Income Maximizing Policy, $\Delta C = 10^5$, $EC = 10^6$

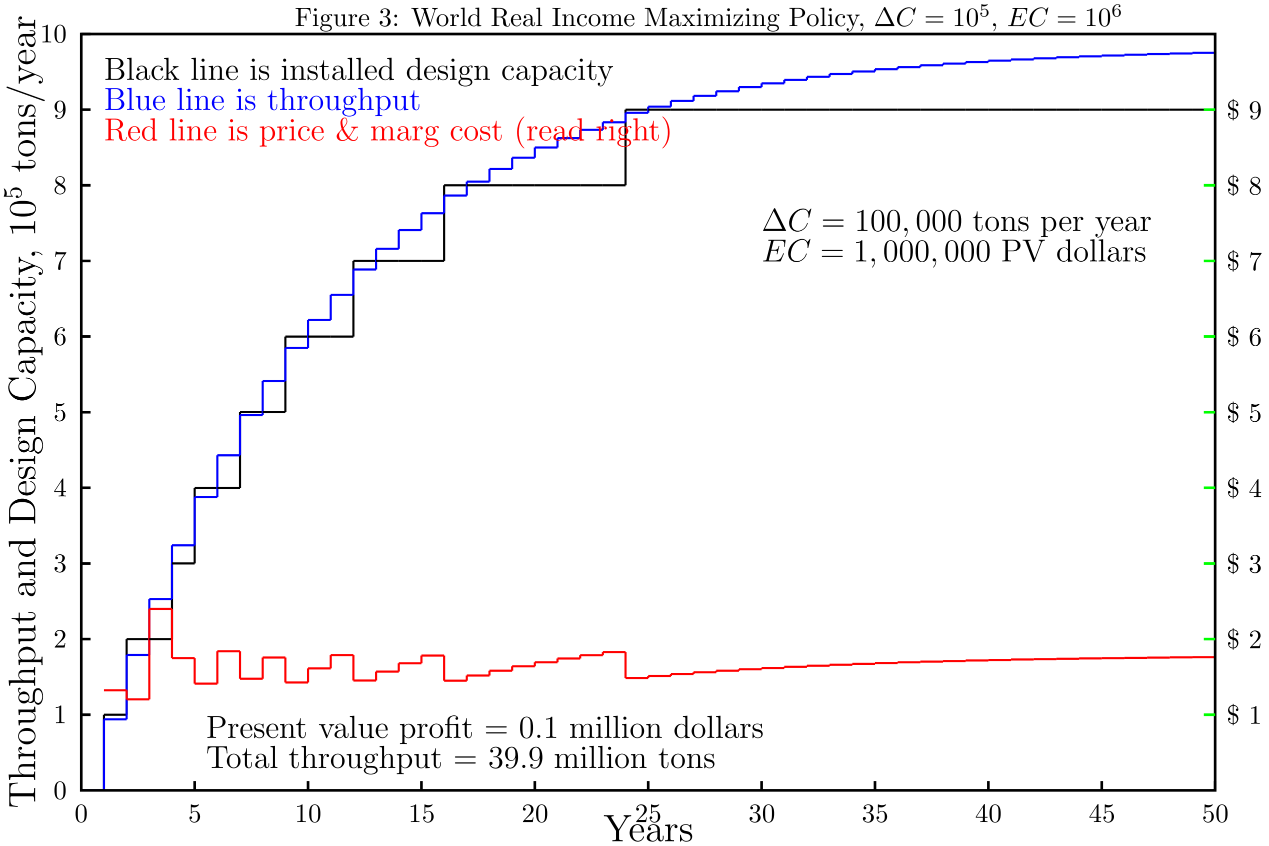


Figure 4: Monopoly Profit Maximizing Policy, $\Delta C = 50,000$, $EC = 10^6$

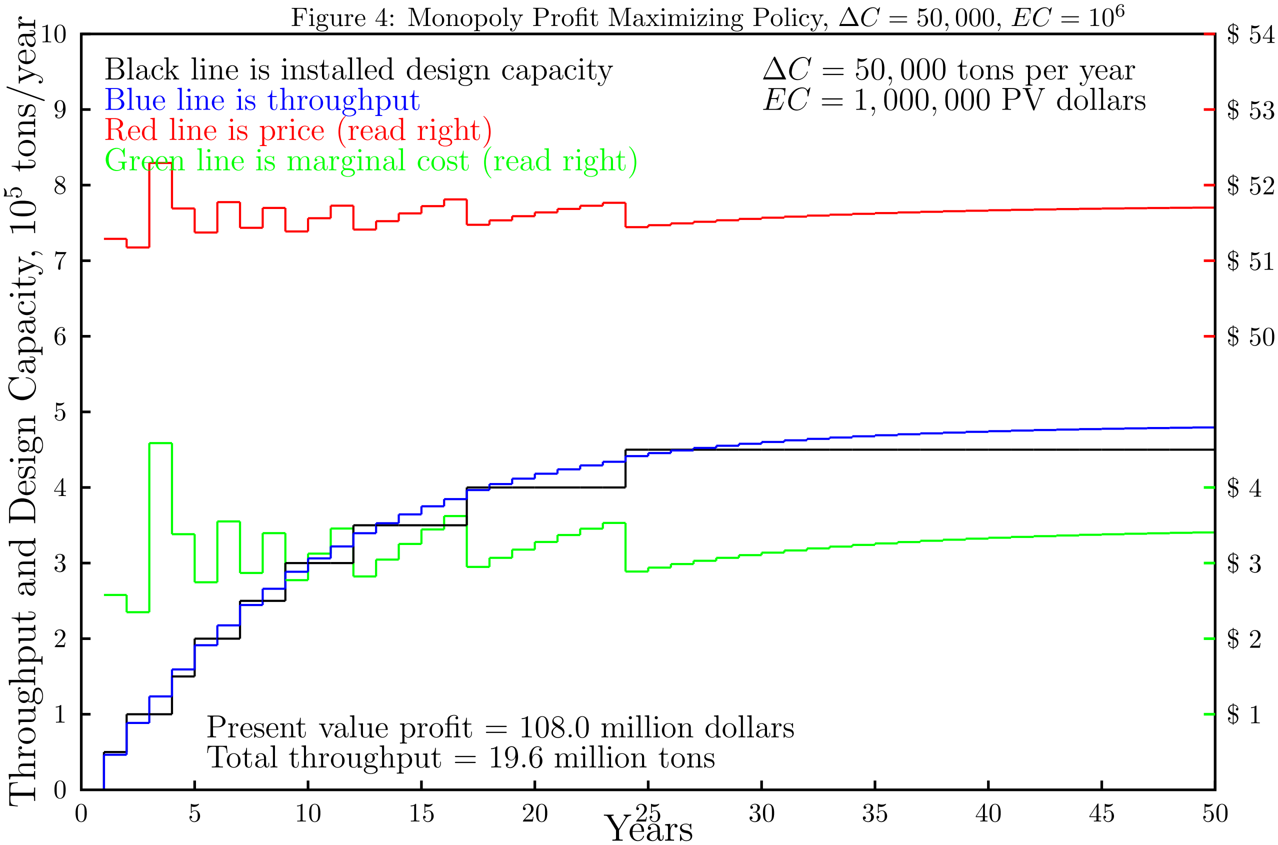


Figure 5: World real Income Maximizing Policy, $\Delta C = 50,000$, $EC = 10^6$

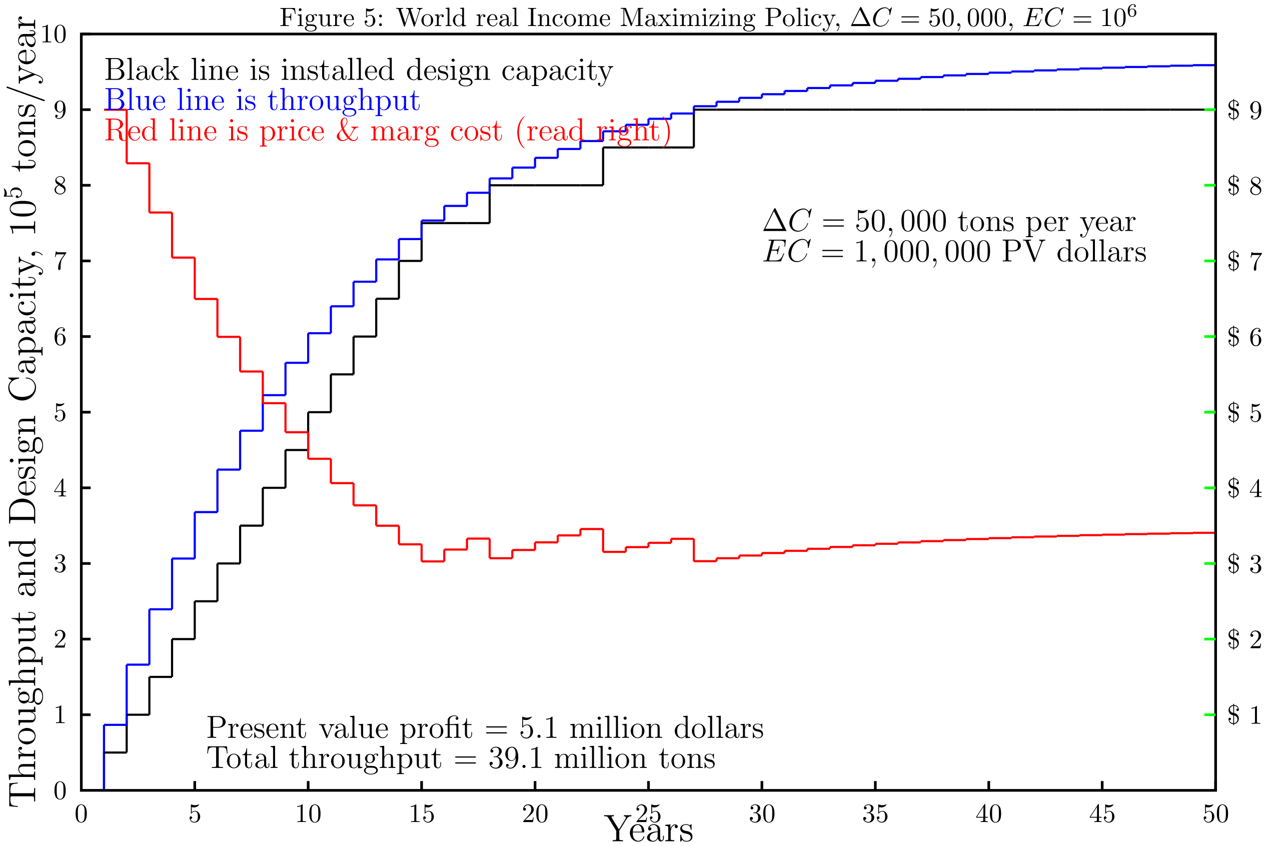


Figure 6: Monopoly Profit Maximizing Policy, $\Delta C = 25,000$, $EC = 10^6$

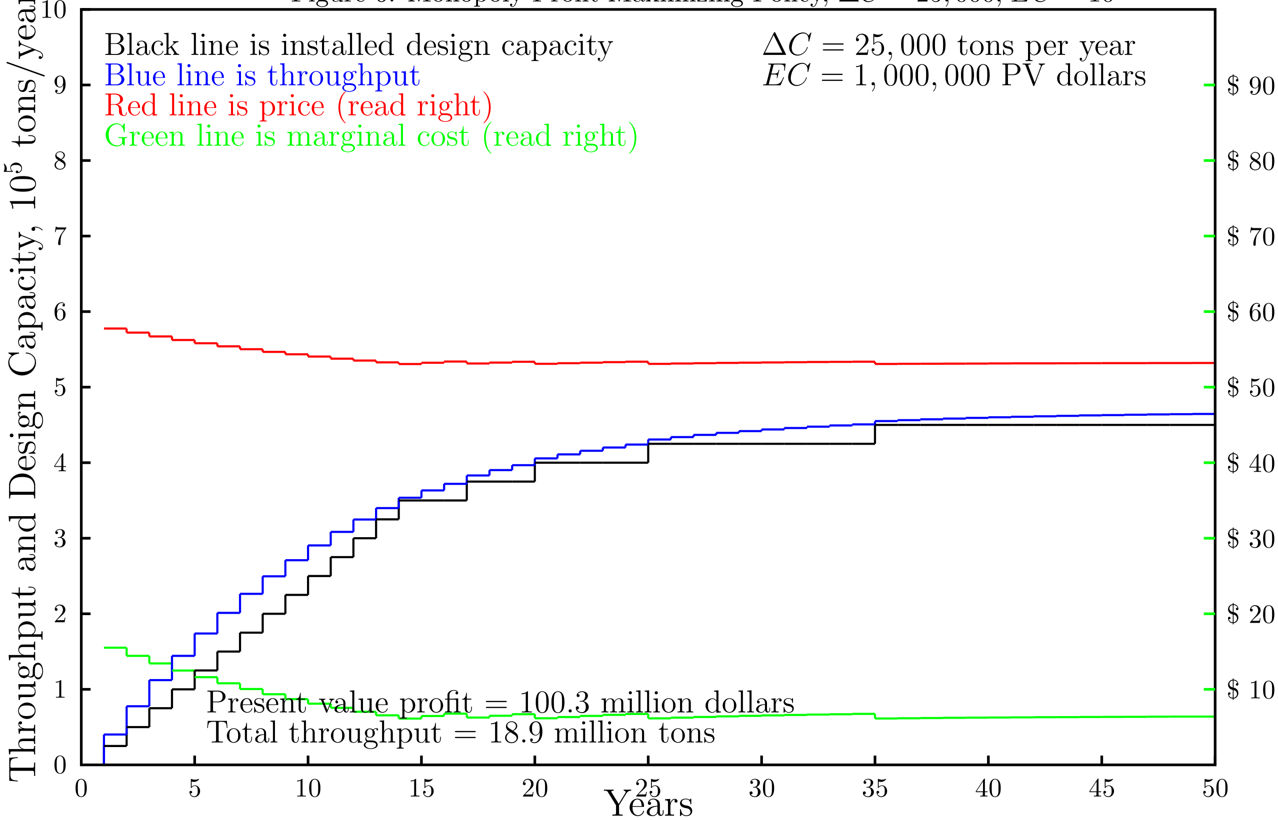


Figure 7: World Real Income Maximizing Policy, $\Delta C = 25,000$, $EC = 10^6$

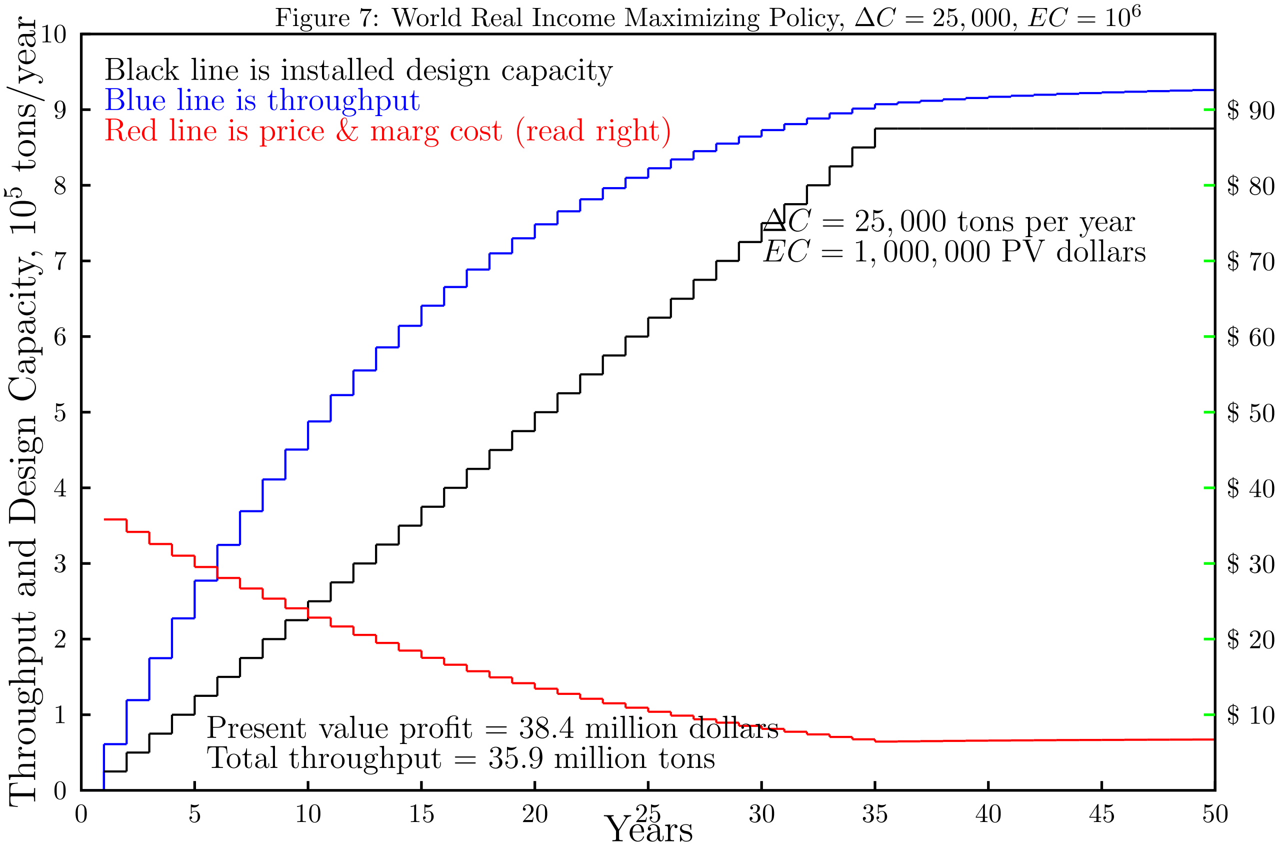


Figure 8: Monopoly Profit Maximizing Policy, $\Delta C = 10^5$, $EC = 10^7$

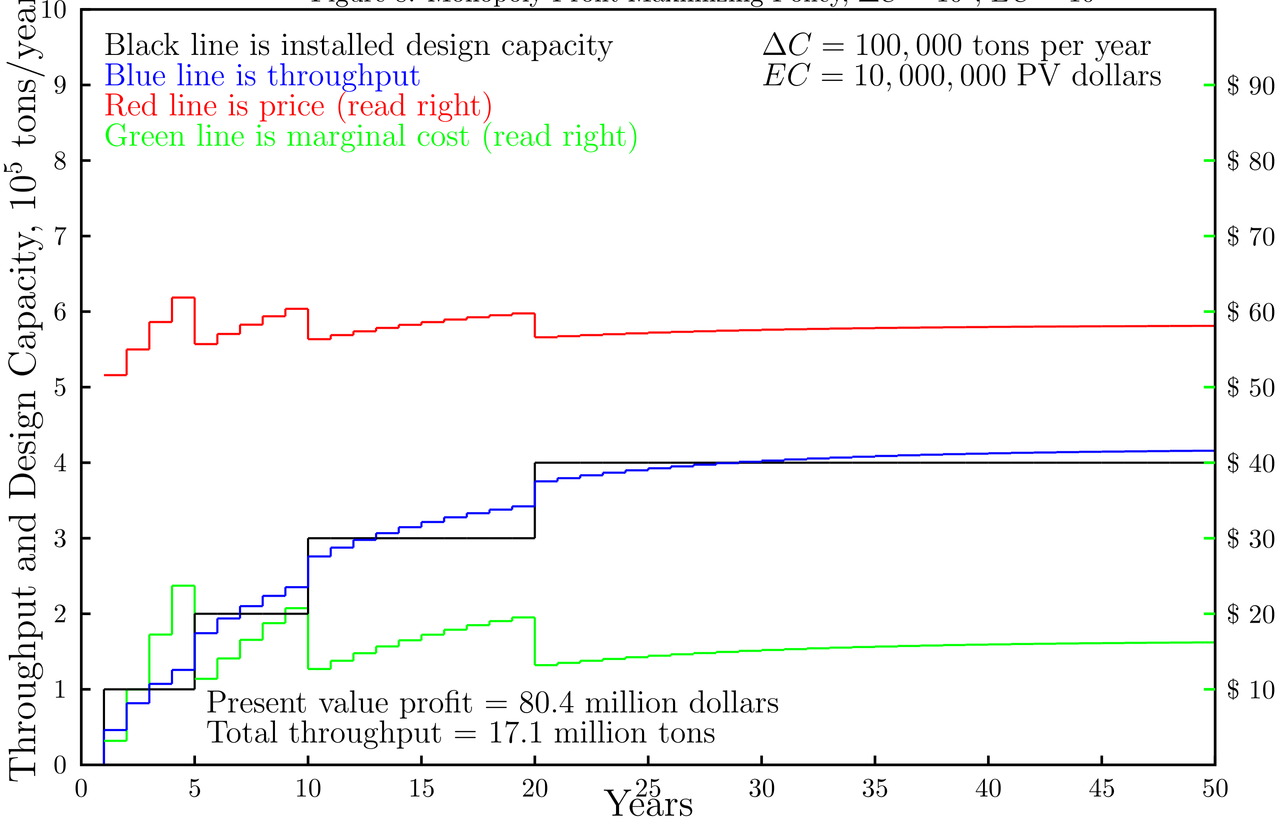


Figure 9: World Real Income Maximizing Policy, $\Delta C = 10^5$, $EC = 10^7$

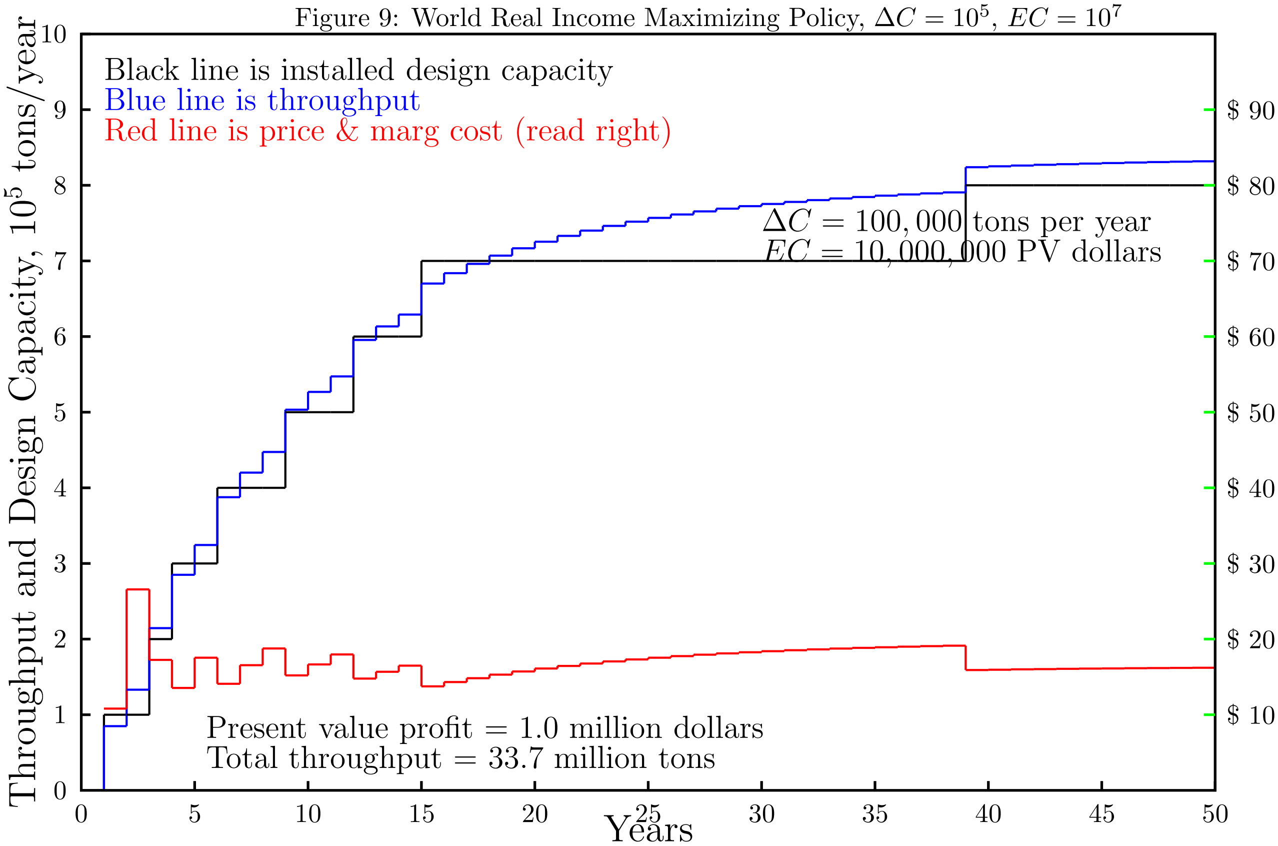


Figure 10: Monopoly Profit Maximizing Policy, $\Delta C = 50,000$, $EC = 10^7$

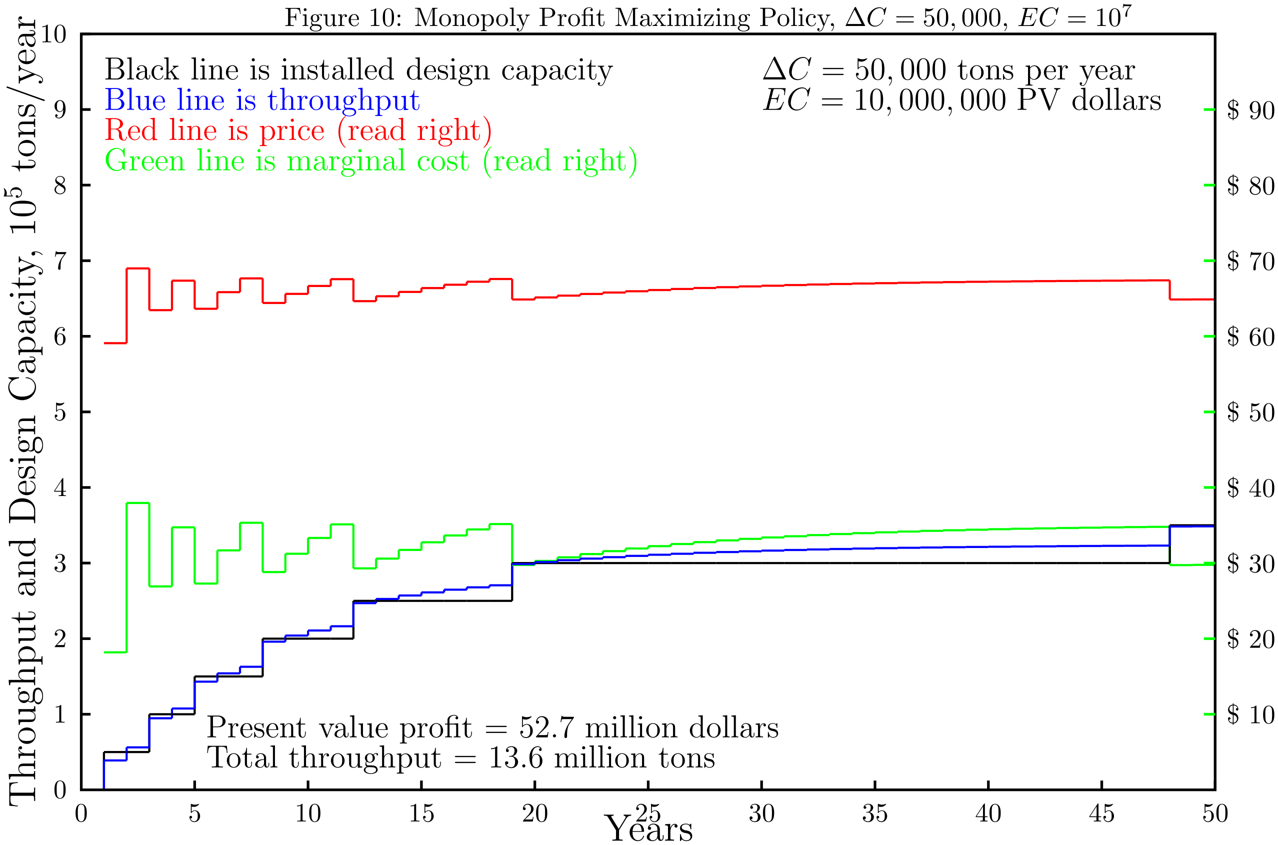
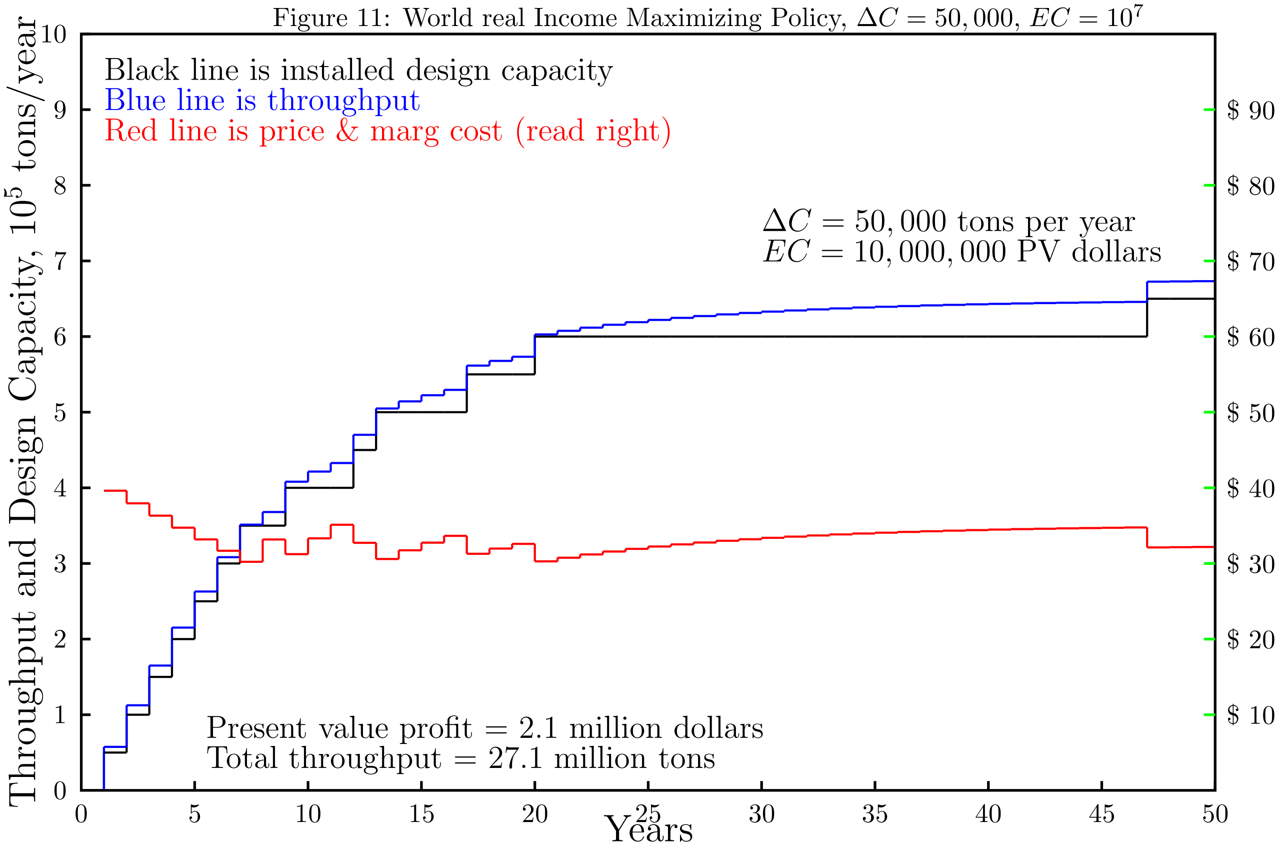


Figure 11: World real Income Maximizing Policy, $\Delta C = 50,000$, $EC = 10^7$



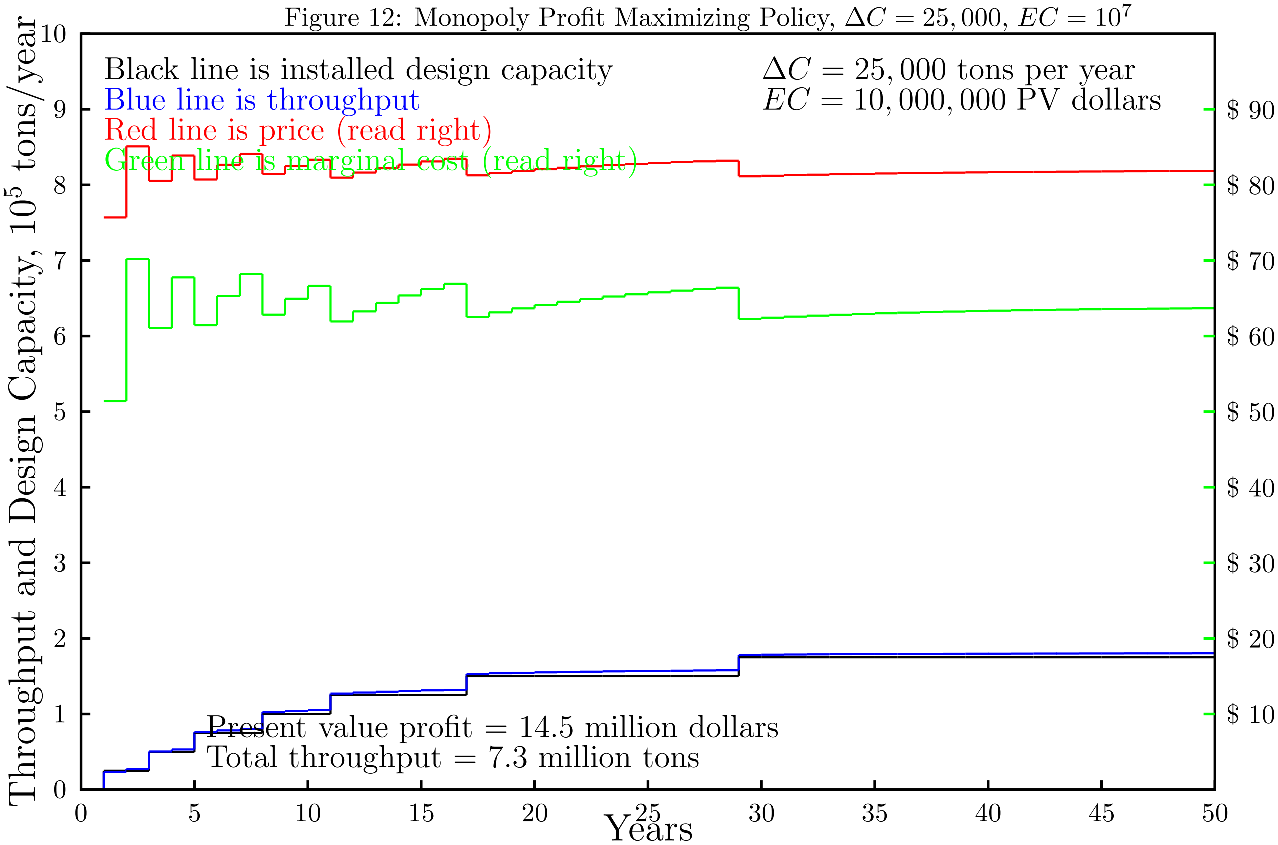


Figure 12: Monopoly Profit Maximizing Policy, $\Delta C = 25,000$, $EC = 10^7$
Black line is installed design capacity
Blue line is throughput
Red line is price (read right)
Green line is marginal cost (read right)
$\Delta C = 25,000$ tons per year
$EC = 10,000,000$ PV dollars
Present value profit = 14.5 million dollars
Total throughput = 7.3 million tons
Throughput and Design Capacity, 10^5 tons/year
Years

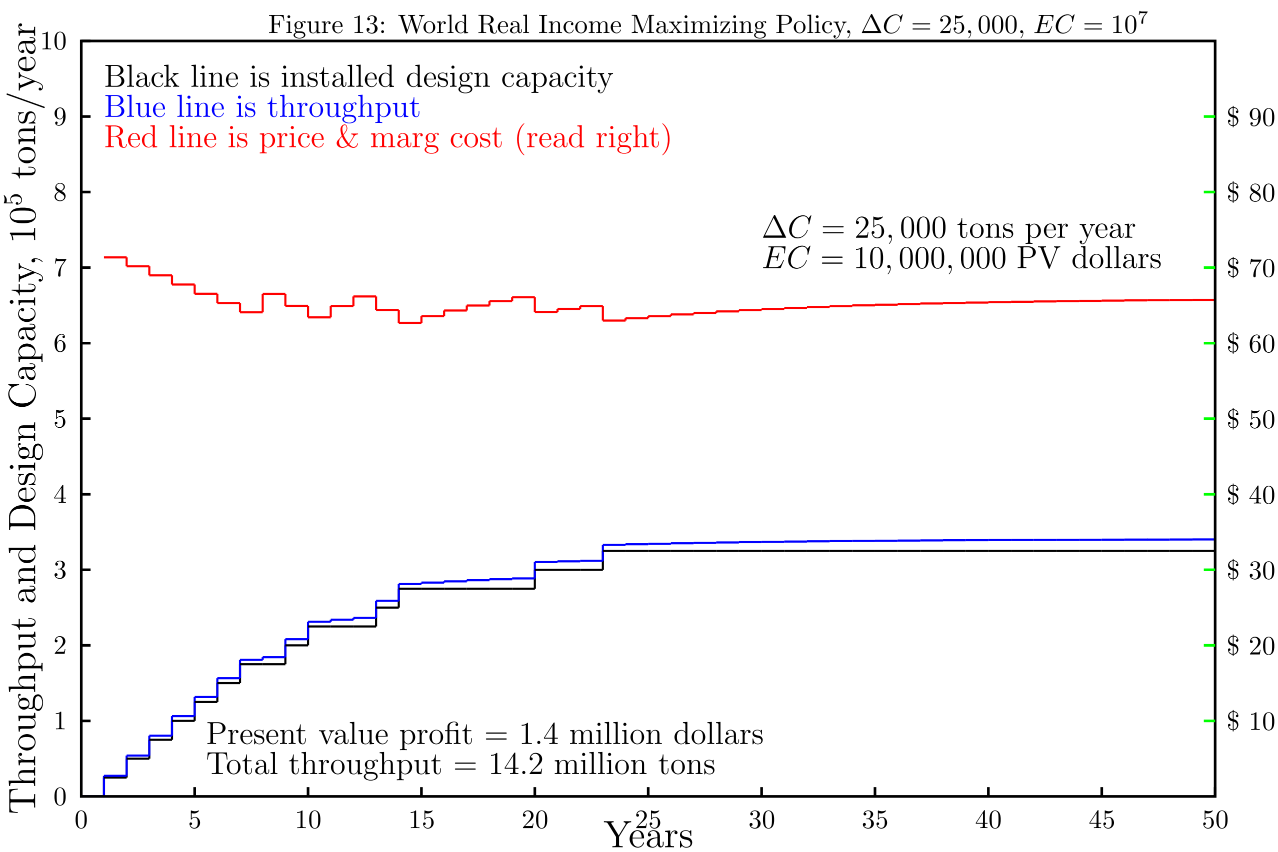


Figure 13: World Real Income Maximizing Policy, $\Delta C = 25,000$, $EC = 10^7$
Black line is installed design capacity
Blue line is throughput
Red line is price & marg cost (read right)
$\Delta C = 25,000$ tons per year
$EC = 10,000,000$ PV dollars
Present value profit = 1.4 million dollars
Total throughput = 14.2 million tons
Throughput and Design Capacity, 10^5 tons/year
Years